**Reading Group on Stochastic Modelling**

*A brief overview of:*

# Incomplete Simultaneous Discrete Response Model with Multiple Equilibria

Tamer (2003)

*Review of Economic Studies*

---

*In relation to:*

# A Structural Model of Dense Network Formation

Mele (2017)

*Econometrica*

# Mele (2017) footnote n.17 p.830

> *"The* **second part of the assumption 1 [see below]** *is an identification restriction, that guarantees the model's* **coherency** *in the sense of Tamer (2003)."*

Individual $i$ **values his popularity** effect as much as $k$ **values the indirect link to** $j$ through any "bridging" individual:

$$\underbrace{w_{kj}^{\theta_v} = w(X_k, X_j|\theta_v)}_{\text{indirect link}} = \underbrace{v(X_k, X_j|\theta_v) = v_{kj}^{\theta_v}}_{\text{popularity}} \quad \forall k, j \in \mathcal{I} \times \mathcal{I}$$

**indirect link:**
utility for $k \neq i,j$
for indirectly linking with $j$
through any $i \neq k,j$

$(k) \rightarrow (i) \rightarrow (j)$

**popularity:**
utility for any $i \neq k,j$
for bridging $k$ to $j$

$(k) \rightarrow (i) \rightarrow (j)$

Under assumption 1.2 any individual $i \in \mathcal{I}$ **internalizes all the externalities generated by his links**:

- The **popularity** component of $U_i(g, \mathbf{X}|\Theta)$ is **equal to** the sum, over all $k \in \mathcal{I} - i$, of the utility of **indirect links of** $k$ **passing through** $i$, which are the indirect links that can be **influenced** by $i$'s link-formation **decisions**;

# Summary of Tamer (2003)

Modelling framework - **2x2 entry-game with perfect information**

two players ($i \in \{-1, 1\}$), action set ($y_i \in \{0, 1\}$) and externalities $\delta_i$. The payoff $\pi_i$ of player $i$ is defined as:

$$\pi_i := y_i(x_i\beta_i + y_{-i}\delta_i + u_i)$$

Where:

- $\mathbf{y} = (y_{-1}, y_1)$ are response variables;
- $\mathbf{x} = (x_{-1}, x_1) \in \mathcal{R}^d$ are observable exogenous variables;
- $\mathbf{u} = (u_{-1}, u_1)$ are random variables unobservable to the econometricians;
- $\beta = (\beta_{-1}, \beta_1, \delta_{-1}, \delta_1)$ are parameters of interest;

Distinction of model identification issues - **Incoherency Vs Incompleteness**

1− **incoherent model**: hasn't a well-defined reduced form, or, is logically inconsistent. For example:
*if externalities $\delta_{-1}$ and $\delta_1$ are both negative, the above model gives*
$Pr[(0,0)|x] + Pr[(0,1)|x] + Pr[(1,0)|x] + Pr[(1,1)|x] > 1$

2− **incomplete model**: the relationship from input variables ($x_i$s and $u_i$s) to responses ($y_i$s) is a **correspondence** and not a function. For example:
*if $\delta_i$s are both negative, $\exists$ a non-empty region of* $\mathbf{u}$'s *support for which the model predicts a non-unique outcome* $(1,0)$ OR $(0,1)$

# Contribution and findings of Tamer (2003)

- **Identifies sufficient conditions for parameter point identification (when externalities have same sign);**
- **Develops a technique for semi-parametric maximum (quasi)likelihood (SML) estimation:** by "replacing" $Pr[(y_{-1}, y_1)|x]$ for outcomes (0,1) and/or (1,0), with local approximations of the the empirical relative frequencies of these outcomes as a function of exogenous variables;

**Why Assumption 1.2 relevant for identification in Mele (2017)?**

1. **externalities are "paired"**: each **indirect-link** effect has a **corresponding popularity effect** with same sign, value and parameter;
2. **number of parameters of the model is reduced**: from 4 $(\theta_u, \theta_m, \theta_w, \theta_v)$ to 3 $(\theta_u, \theta_m, \theta_v)$. Condition necessary for model completeness (?);
3. guarantees that the system of **conditional linking probabilities** implied by the model **generate a proper joint distribution** of the network matrix;
4. can **use** the potential function $\mathcal{Q}$ **to construct the network as a best-response potential game**. Via **sequential** link-formation **decisions** the game converges through an **improvement path** to a **Pure Strategy Nash Equilibrium** network with $Pr = 1$;

# References I


- Jackson, M. O. (2008), Social and Economics Networks. Princeton
- Lovasz, L. (2012), Large Networks and Graph Limits. American Mathematical Society Colloquium Publications, Vol. 60. American Mathematical Society

- Badev, A. (2017), "Discrete Games in Endogenous Networks: Equilibria and Policy", Workin Paper. Available at arXiv https://arxiv.org/abs/1705.03137v1
- Besag, J. (1974), "Spatial Interaction and the Statistical Analysis of Lattice Systems", Journal of the Royal Statistical Society. Series B (Methodological), 36(2):192–236.
- Caimo, A., & N. Friel (2011), "Bayesian Inference for Exponential Random Graph Models", Social Networks, 33(1):41–55
- Chatteriee, S., & S. R. S. Varadhan (2011), "The Large Deviation Principle for the Erdös-Rényi Random Graph" European Journal of Combinatorics, 32(7):1000–1017. Homomorphisms and Limits
- Christakis, N., J. Fowler, G. W. Imbens, & K. Kalyanaraman (2010), "An Empirical Model for Strategic Network Formation" Harvard University
- Frank, O., & D. Strauss (1986), "Markov Graphs", Journal of the American Statistical Association, 81:832–842

# References II


- Graham, B. (2017), "An Econometric Model of Network FormationWith Degree Heterogeneity", Econometrica, 85(4):1033-1063
- Gouriéroux, J., Lafont, J. J. and Monfort, A. (1981), "Coherency Conditions in Simultaneous Linear Equation Models with Endogenous Switching Regimes", Econometrica, 48(3):675–696.
- Hsieh, C.-S., & L.-F. Lee (2016), "A Social Interactions Model With Endogenous Friendship Formation and Selectivity", Journal of Applied Econometrics, 31(2):301–319
- Leung, M. (2018), "A weak law for moments of pairwise-stable networks", Working Paper. Available at SSRN: https://ssrn.com/abstract=2663685
- Monderer, D., & L. Shapley (1996), "Potential Games", Games and Economic Behavior, 14, 124–143.
- Mele, A., & L. Zhu (2017), "Approximate Variational Estimation for a Model of Network Formation", Working Paper. Available at arXiv: https://arxiv.org/abs/1702.00308
- Menzel, K. (2017), "Strategic Network Formation With Many Agents", Working Papers, NYU
- Murray, I. A., Z. Ghahramani, & D. J. C. MacKay (2006), "MCMC for Doubly-Intractable Distributions", Uncertainty in Artificial Intelligence
- Miyauchi, Y. (2016), "Structural Estimation of a Pairwise Stable Network Formation With Nonnegative Externality", Journal of Econometrics, 195(2):224–235

# References III


- Snijders, T. A. B. (2002), "Markov Chain Monte Carlo Estimation of Exponential Random Graph Models", Journal of Social Structure, 3(2)
- Tamer, E. (2003), "Incomplete Simultaneous Discrete Response Model with Multiple Equilibria", The Review of Economic Studies, 70:147–165.
- Wainwright, M. J., & M. Jordan (2008), "Graphical Models, Exponential Families, and Variational Inference", Foundations and Trends© in Machine Learning, 1(1–2):1–305