Reading Group on Stochastic Modelling

A brief overview of:

Incomplete Simultaneous Discrete Response Model with Multiple Equilibria

Tamer (2003)

Review of Economic Studies

In relation to:

A Structural Model of Dense Network Formation

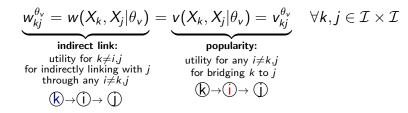
Mele (2017)

Econometrica

Mele (2017) footnote n.17 p.830

"The second part of the assumption 1 [see below] is an identification restriction, that guarantees the model's coherency in the sense of Tamer (2003)."

Individual *i* values his popularity effect as much as *k* values the indirect link to *j* through any "bridging" individual:



Under assumption 1.2 any individual $i \in \mathcal{I}$ internalizes all the externalities generated by his links:

• The **popularity** component of $U_i(g, X | \Theta)$ is **equal to** the sum, over all $k \in \mathcal{I} - i$, of the utility of **indirect links of** k **passing through** i, which are the indirect links that can be **influenced** by i's link-formation **decisions**;

Summary of Tamer (2003)

Modelling framework - 2x2 entry-game with perfect information

two players ($i \in \{-1, 1\}$), action set ($y_i \in \{0, 1\}$) and externalities δ_i . The payoff π_i of player *i* is defined as:

Where:

$$\pi_i := y_i (x_i \beta_i + y_{-i} \delta_i + u_i)$$

- $\mathbf{y} = (y_{-1}, y_1)$ are response variables;
- $\circ \mathbf{x} = (x_{-1}, x_1) \in \mathcal{R}^d$ are observable exogenous variables;
- \circ **u** = (u_{-1} , u_1) are random variables unobservable to the econometricians;
- $\beta = (\beta_{-1}, \beta_1, \delta_{-1}, \delta_1)$ are parameters of interest;

Distinction of model identification issues - Incoherency Vs Incompleteness

- 1- **incoherent model**: hasn't a well-defined reduced form, or, is logically inconsistent. For example: if externalities δ_{-1} and δ_1 are both negative, the above model gives Pr[(0,0)|x] + Pr[(0,1)|x] + Pr[(1,0)|x] + Pr[(1,1)|x] > 1
- 2- **incomplete model**: the relationship from input variables $(x_i \text{s and } u_i \text{s})$ to responses $(y_i \text{s})$ is a **correspondence** and not a function. For example: *if* $\delta_i \text{s are both negative, } \exists$ *a non-empty region of* **u**'s *support for which the model predicts a non-unique outcome* (1,0) OR (0,1)

Contribution and findings of Tamer (2003)

- Identifies sufficient conditions for parameter point identification (when externalities have same sign);
- Develops a technique for semi-parametric maximum (quasi)likelihood (SML) estimation: by "replacing" Pr[(y-1, y1)|x] for outcomes (0,1) and/or (1,0), with local approximations of the the empirical relative frequencies of these outcomes as a function of exogenous variables;

Why Assumption 1.2 relevant for identification in Mele (2017)?

- 1. externalities are "paired": each indirect-link effect has a corresponding popularity effect with same sign, value and parameter;
- 2. number of parameters of the model is reduced: from 4 (θ_u , θ_m , θ_w , θ_v) to 3 (θ_u , θ_m , θ_v). Condition necessary for model completeness (?);
- 3. guarantees that the system of **conditional linking probabilities** implied by the model **generate a proper joint distribution** of the network matrix;
- 4. can use the potential function Q to construct the network as a best-response potential game. Via sequential link-formation decisions the game converges through an improvement path to a Pure Strategy Nash Equilibrium network with Pr = 1;

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