

Reading Group on Stochastic Modelling

<https://matteoiacopini.github.io/stochmodgroup/>

A Structural Model of Dense Network Formation

Mele (2017)

Econometrica






Introduction

Literature






Related works

- ▶ similar paper [6]
- ▶ extensions [12], [3], [9]


Model

-  potential games [11]
-  network formation models [1]
-  ERGM theory [7], [16]

Asymptotics

-  many networks asymptotics [15]
-  large network asymptotics [8], [13], [10]
 -  graph limits [2]
 -  large deviations [5]
 -  variational methods for the exponential family [17]

Estimation

-  ERGM estimation [16], [4]

Introduction

Strategic models of network formation provide a framework to interpret the observed network as the equilibrium of a (potential) game.

Estimation and identification of strategic models is challenging

- 1) **multiple equilibria** \Rightarrow links generate externalities not fully accounted for by agents
- 2) **curse dimensionality** \Rightarrow # network configs grows exponentially with # agents
- 3) **data on single graph** \Rightarrow only one network snapshot is observable

Proposed model of network formation

- ❖ combines features from the **strategic** and **random network** formation literature
- ❖ players' utilities depend on payoffs from **direct links** and link **externalities** (e.g., reciprocity, indirect friends, popularity, ...)
- ❖ network formation is dynamic: each period, a player meets another one and decides whether to form a new link, keep an existing link, or do nothing
- ❖ process generates a **sequence of directed dense** graphs

Model of Network Formation

Model of Network Formation

Setup

- n agents, with characteristics $X_i \in \mathbb{R}^A$, $\forall i \in \mathcal{I} := \{1, \dots, n\}$
- discrete time $t \in \mathbb{N}$
- directed, binary network $G \in \mathcal{G}$, realisations each time g^t

Definition 1 (Individual utility function).

Let $u_{ij}^{\theta_u} = u(X_i, X_j | \theta_u)$, $m_{ij}^{\theta_m} = m(X_i, X_j | \theta_m)$, $v_{ij}^{\theta_v} = v(X_i, X_j | \theta_v)$, $w_{ij}^{\theta_w} = w(X_i, X_j | \theta_w)$ where $\theta = (\theta_u, \theta_m, \theta_v, \theta_w)' \in \mathbb{R}^4$ are parameters.

The **utility of agent i** from network g is given by the sum of four components

$$U_i(g, X | \theta) = \underbrace{\sum_{j=1}^n g_{ij} u_{ij}^{\theta_u}}_{\text{direct links}} + \underbrace{\sum_{j=1}^n g_{ij} g_{ji} m_{ij}^{\theta_m}}_{\text{mutual links}} + \underbrace{\sum_{j=1}^n g_{ij} \sum_{\substack{k=1 \\ k \neq i, j}}^n g_{jk} v_{ik}^{\theta_v}}_{\text{indirect links}} + \underbrace{\sum_{j=1}^n g_{ij} \sum_{\substack{k=1 \\ k \neq i, j}}^n g_{ki} w_{kj}^{\theta_w}}_{\text{popularity}}.$$

► “**Markovian**” only indirect links are valuable and are perfect substitutes (no utility from two-links-away contacts)

Potential Game

Definition 2 (Potential Game).

A game is said to be a **Potential Game** if the incentive of all players to change their strategy (here: link formation choice) can be expressed using a single global function called the **potential function** $\mathcal{Q} : \mathcal{G} \times \mathcal{X} \rightarrow \mathbb{R}$ such that:

$$\mathcal{Q}(g_{ij}, g_{-ij}, X) - \mathcal{Q}(g'_{ij}, g_{-ij}, X) = U_i(g_{ij}, g_{-ij}, X) - U_i(g'_{ij}, g_{-ij}, X), \quad \forall i, j \forall g_{-ij}$$

Remark 1.

The **potential function** is useful for:

- analyse **equilibrium properties** of games,
set of pure Nash equilibria corresponds to the local optima of potential function;
existence of profitable deviations performed using the potential, instead of checking each player's possible deviation
- study **convergence** and finite-time convergence of iterated game towards a Nash equilibrium

Assumption 1. (Preferences)

$$m_{ij}^{\theta_m} = m(X_i, X_j | \theta_m) = m(X_j, X_i | \theta_m) = m_{ji}^{\theta_m} \quad \forall i, j \in \mathcal{I} \times \mathcal{I}$$

$$w_{kj}^{\theta_v} = w(X_k, X_j | \theta_v) = v(X_k, X_j | \theta_v) = v_{kj}^{\theta_v} \quad \forall k, j \in \mathcal{I} \times \mathcal{I}$$

- ▶ first is **necessary for identification** of the utility from indirect links and popularity;
- ▶ second makes another agent i **internalise the externality** she creates.

Proposition 1 (Existence Potential Function).

Under Assumption 1, the deterministic components of the incentives of any player in any state of the network are summarized by a **potential function** $Q : \mathcal{G} \times \mathcal{X} \rightarrow \mathbb{R}$ and the network game is a **Potential Game**

$$Q(g, X | \theta) = \sum_{i=1}^n \sum_{j=1}^n g_{ij} u_{ij}^{\theta_u} + \sum_{i=1}^n \sum_{j>i}^n g_{ij} g_{ji} m_{ij}^{\theta_m} + \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n \sum_{\substack{k=1 \\ k \neq i, j}}^n g_{ij} g_{jk} v_{ik}^{\theta_v}$$

Q is an aggregate function summarising: (i) state of network; (ii) deterministic incentives of players in each state.

Network formation process

Stochastic best-response dynamics, generating a Markov chain of graphs:

- for each t , randomly chosen player i meets j according to meeting technology
- *meeting process* is a stochastic sequence $\mathbf{m} = \{m^t\}_t$ supported on $\mathcal{I} \times \mathcal{I}$, with realisations $m^t = ij = \{i, j\}$ whose probability is

$$\mathbb{P}(m^t = ij | g^{t-1}, X) = \rho(g^{t-1}, X_i, X_j)$$

Assumption 2. (Meeting process)

The meeting probability between i, j does not depend on the existence of a link between them, and each meeting has a positive probability of occurring, that is

$$\rho(g^{t-1}, X_i, X_j) = \rho(g_{-ij}^{t-1}, X_i, X_j) > 0 \quad \forall ij$$

► guarantees any equilibrium network can be reached with positive probability

Idea: Assumption 2 makes the Markov chain irreducible

► identification: if ρ depends on link $g_{ij} \Rightarrow$ prevents closed form likelihood

Players' rules

- conditional on meeting $m^t = ij$, player i updates link g_{ij} to maximise her utility
- existing network g_{-ij}^{t-1} is taken as given
- **complete information**: everybody known each others' attributes and whole network
- **myopia**: agents not account for effects of their linking strategy on future evolution of network

Assumption 3. (Idiosyncratic shocks)

Idiosyncratic shock on individual preferences: $\varepsilon_{ij,t} \stackrel{iid}{\sim} EV_1(\varepsilon_{ij,t})$ Type I extreme value distribution, *iid* among links and across time

$\Rightarrow > 0$ proba moving out from any state \rightarrow eliminates absorbing states

- link established if and only if

$$U_i(g_{ij}^t = 1, g_{-ij}^{t-1}, X|\theta) + \varepsilon_{1t} > U_i(g_{ij}^t = 0, g_{-ij}^{t-1}, X|\theta) + \varepsilon_{0t}$$

Process generates a Markov chain of networks:

- ✓ transition proba determined by: (i) meeting process, (ii) agents' linking choices
- ✓ irreducible (from Assumption 2), aperiodic (from Assumption 3)

Equilibrium

Remark 2.

Any change in utility for any agent is equivalent to change in potential Q . So, any deviation from Nash (equilibrium) network must decrease the potential.

Thus, the Nash network is a local maximizer of the potential function over the set of networks that differ from the current network for at most one link.

Theorem 1 (Uniqueness and Characterisation of Stationary Equilibrium).

The network formation game, under Assumptions 1–3, converges to a **unique** stationary distribution

$$\pi(g, X|\theta) = \frac{\exp\{Q(g, X|\theta)\}}{\sum_{\omega \in \mathcal{G}} \exp\{Q(\omega, X|\theta)\}} \quad (1)$$

Comments

- existence and uniqueness come from irreducibility and aperiodicity of Markov chain
 - closed form **stationary** $\pi(g, X|\theta)$ corresponds to the **likelihood** of observing a specific network configuration in the long run
 - **estimation**: uniqueness avoids multiple equilibria
 \Rightarrow unique stationary = unique likelihood
- \Rightarrow can **estimate** θ with **only one network**, assumed drawn from stationary equilibrium
- $\pi(g, X|\theta)$ coincides with **likelihood** of **ERGM** (Exponential Random Graph Model), where probability observing a network is proportional to exponential of linear combination of network statistics

Corollary 2.1.

Let Assumptions 1–3 hold. If the utility functions are linear in parameters, the stationary distribution $\pi(g, X|\theta)$ describes an ERGM, with $\mathbf{t}(g, X)$ a vector of canonical statistics

$$\pi(g, X|\theta) = \frac{\exp\{\theta' \mathbf{t}(g, X)\}}{\sum_{\omega \in \mathcal{G}} \exp\{\theta' \mathbf{t}(\omega, X)\}} \quad (2)$$

Model without shocks

Proposition 2 (Model Without Shocks: Equilibria and Long Run).

Consider the model without idiosyncratic preference shocks. Under Assumptions 1-2:

- (i) there exists at least one pure-strategy Nash equilibrium network.
- (ii) the set $\mathcal{NE}(\mathcal{G}, X, U)$ of all pure-strategy Nash equilibria of the network formation game is completely characterized by the local maxima of the potential function:

$$\mathcal{NE}(\mathcal{G}, X, U) = \left\{ g^* : g^* = \arg \max_{g \in \mathcal{N}(g^*)} Q(g, X) \right\}.$$

- (iii) any pure-strategy Nash equilibrium is an absorbing state.
- (iv) as $t \rightarrow \infty$, the network converges to one of the Nash networks with probability 1.

Extensions

Utility functions

👉 possible to include **additional utility components**, as long as possible to find **restrictions on payoffs** that guarantee the existence of a potential function

Undirected networks

👉 possible to extend existence results, characterisation of equilibrium, relation with ERGM and asymptotic results to **undirected networks**

Sparsity

👉 model with **negative linking externalities** is compatible with a certain degree of sparsity

Estimation and Identification

Estimation and Identification

Likelihood function

$$L(g, X|\theta) = \pi(g, X|\theta) = \frac{Q(g, X|\theta)}{\sum_{\omega \in \mathcal{G}} Q(\omega, X|\theta)} = \frac{Q(g, X|\theta)}{c(\mathcal{G}, X, \theta)}$$

whose normalizing constant $c(\mathcal{G}, X, \theta)$ is **intractable** since it sums $2^{n(n-1)}$ terms.

✗ standard ML infeasible

✗ MCMC with standard MH step infeasible (ratio of normalizing constants)

Estimation Algorithm

ERGM literature \Rightarrow approximate $c(\mathcal{G}, X, \theta)$ via MCMC (for fixed θ_0)

Algorithm 1 Metropolis-Hastings for Network Simulations

- 1: **procedure** MH_NETSIM(θ_0, g_0, R)
- 2: **for** $r = 1, \dots, R$ **do**
- 3: 1) propose network $g' \sim q_g(g'|g^{(r)})$
- 4: 2) accept network g' with probability

$$\alpha(g^{(r)}, g') = \min \left\{ 1, \frac{\exp\{Q(g', X|\theta_0)\} q_g(g^{(r)}|g')}{\exp\{Q(g^{(r)}, X|\theta_0)\} q_g(g'|g^{(r)})} \right\}$$

- 5: **end for**
 - 6: **return** sequence of R networks $\{g^{(r)}\}_r$
 - 7: **end procedure**
-

✓ not requires $c(\mathcal{G}, X, \theta)$

✗ *slow convergence*

✗ *local sampler* at each iteration, update link g_{ij} according to $\alpha(\cdot, \cdot)$

✗ *degeneracy* problem: large probability mass on few networks

Estimation Algorithm

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► how to choose $q_g(\cdot | g^{(r)})$?

Asymptotic behaviour

Classes of asymptotics for networks

- 1) *many networks* \Rightarrow same players, growing number of **networks**
- 2) large networks \Rightarrow growing number of **players**, same network

- ▶ Hp: **homogeneous** players (i.e. $X_i = X_j, \forall i, j$)
- ▶ potential function re-scaled by $n^{\nu(H)}$, with $\nu(H)$ # players in each utility term
- ▶ example **re-scaled likelihood**, with $\mathcal{T}(g) = \alpha t(H_1, g) + \beta t(H_2, g)$ re-scaled potential and $\psi_n = n^{-2} \log(c(\alpha, \beta, \mathcal{G}_n))$ log-normalising constant

$$\begin{aligned} \pi_n(g|\alpha, \beta) &= \frac{\exp \left\{ n^2 \left[\alpha \frac{\sum_{i=1}^n \sum_{j=1}^n g_{ij}}{n^2} + \beta \frac{\sum_{i=1}^n \sum_{j=1}^n \sum_{k \neq i}^n g_{ij} g_{jk}}{n^3} \right] \right\}}{c(\alpha, \beta, \mathcal{G}_n)} \\ &= \exp \left\{ n^2 [\mathcal{T}(g) - \psi_n] \right\} \end{aligned} \quad (3)$$

Asymptotic behaviour

Theorem 2 (Nonnegative Link Externalities).

Model (3) with **nonnegative link externalities** $\beta \geq 0$ exhibits the following behaviour

- 1) **asymptotic normalizing constant** ψ solves

$$\psi = \lim_{n \rightarrow \infty} \psi_n = \max_{\mu \in [0,1]} \left\{ \alpha\mu + \beta\mu^2 - \mu \log(\mu) - (1 - \mu) \log(1 - \mu) \right\} \quad (4)$$

- 2) networks generated by the model are **indistinguishable from directed Erdős–Rényi** graph with linking probability μ^* , defined as follows:

- (a) if the maximization (4) has a unique solution, then μ^* satisfies $2\beta\mu(1 - \mu) < 1$ for almost all $\alpha \in \mathbb{R}$ and $\beta \geq 0$, and solves

$$\mu = \frac{\exp \{ \alpha + 2\beta\mu \}}{1 + \exp \{ \alpha + 2\beta\mu \}} \quad (5)$$

- (b) if the maximization (4) has two solutions, then μ^* picked randomly from same proba distribution over μ_1^* and μ_2^* , such that $\mu_1^* < 0.5 < \mu_2^*$, and both solve (5) and satisfy $2\beta\mu(1 - \mu) < 1$.

Comments on Theorem 2

- ▶ consistent estimator of log-normalising constant – analogue of variational representation of the discrete exponential family
- ▶ $\beta \geq 0 \Rightarrow$ realisations using (α, β) indistinguishable from those using $(\alpha', 0) = (\log(\mu^*/(1 - \mu^*)), 0)$, that is from Erdős-Rényi model

Corollary 2.2.

When $\beta \geq 0$, the **externality cannot be identified**.

Corollary 2.3.

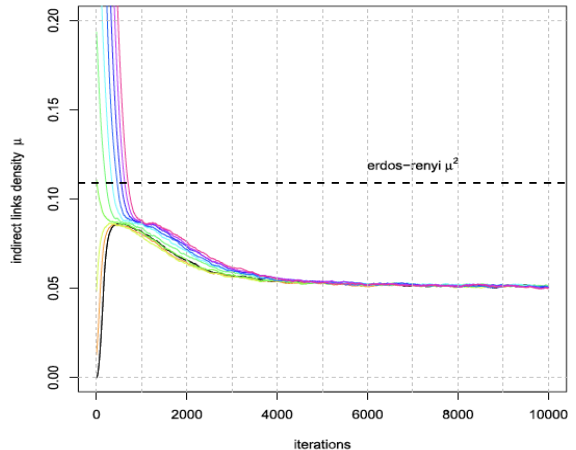
When $\beta \geq 0$, Algorithm 1 is not necessary since Erdős-Rényi graphs can be simulated using Bernoulli draws.

Asymptotic behaviour

Theorem 3 (Negative Link Externalities).

If $\beta < 0$ and sufficiently large in magnitude, model (3) is asymptotically different from a directed Erdős-Rényi model.

✓ sparser graphs than Erdős-Rényi



Asymptotic behaviour

Theorem 3 (Negative Link Externalities).

If $\beta < 0$ and sufficiently large in magnitude, model (3) is asymptotically different from a directed Erdős-Rényi model.

▶ how much “sufficiently large” magnitude?

▶ how to know it, since we must estimate β ?

Asymptotic behaviour

Consider an additional utility component (cyclic triangles):

$$T(g) = \alpha t(H_1, g) + \beta t(H_2, g) + \gamma t(H_3, g), \quad t(H_3, g) = \sum_{i=1}^n \sum_{j=1}^n \sum_{k \neq i}^n g_{ij} g_{jk} g_{ki} \quad (6)$$

Theorem 4.

Consider model (6) as $n \rightarrow \infty$

1) If $\beta \geq 0$ and $\gamma \geq 0$, then the asymptotic normalising constant ψ solves

$$\psi = \lim_{n \rightarrow \infty} \psi_n = \max_{\mu \in [0,1]} \left\{ \alpha \mu + \beta \mu^2 + \gamma \mu^3 - \mu \log(\mu) - (1 - \mu) \log(1 - \mu) \right\} \quad (7)$$

and model is asymptotically indistinguishable from directed Erdős-Rényi graph, with μ^* maximising (7). If the maximisation problem has multiple solutions, then μ^* picked randomly from some distribution on maximisers.

2) If at least **one externality is negative** (i.e. $\beta < 0$ or $\gamma < 0$) and sufficiently large, then model (6) not converge asymptotically to directed Erdős-Rényi graph and **externalities can be identified**.

Summary of Asymptotics

Remark 3.

Homogeneous players ($X_i = X_j, \forall i, j$):

(a) positive externalities

- asymptotically indistinguishable from Erdős-Rényi graph
- externalities not identified
- can **approximate likelihood** of model via likelihood of Erdős-Rényi graph

(b) at least one externality negative and large

- asymptotically **sparser** than Erdős-Rényi graph
- externalities **identified**

Heterogeneous players ($\exists i, j$ such that $X_i \neq X_j$):

- no results
- preliminary study in Mele & Zhu (2017) - working paper

Sampler Convergence

Theorem 5 (Convergence of Local Sampler with Nonnegative Externalities).

Model (6), with probability of meeting $\rho_{ij} = 1/(n(n-1))$. Fix $\gamma \geq 0$. Then, in the case of nonnegative externalities $\beta \geq 0$, there exists a V-shaped region of the parameter space delimited by functions $S_\gamma(\phi_1(\alpha)), S_\gamma(\phi_2(\alpha))$ such that

- 1) if (α, β) belongs to the V-shaped region, then model converges to stationarity in e^{Cn^2} steps, $C > 0$. This results holds for any local sampler.
- 2) otherwise, model converges in $Cn^2 \log(n)$ steps, $C > 0$.

Intuition:

- (1a) in the V-shaped region problem (7) has 2 *local maxima*, the sampler spend exponential time at one of them (i.e. probability e^{-Cn^2} to escape from local max)
- (1b) increasing $\gamma \implies$ increase area of exponentially slow convergence
- (2a) when convergence is quadratic \implies sampler feasible for $n < 500$
- (2b) this happens when model is indistinguishable from directed Erdős-Rényi graph

Simulation and Estimation

Simulation and Estimation in Finite Networks

Posterior inference via approx version of exchange algorithm of MGM06 [14]

- ▶ double Metropolis-Hastings step to avoid computing $c(\mathcal{G}, X, \theta)$
- ▶ data augmentation via auxiliary network g'
- ▶ higher $R \implies$ better approximation of posterior, but higher rejection rate

Algorithm 2 Approximate Exchange Algorithm

- 1: **procedure** AEA(θ, g, M, R)
- 2: **for** $m = 1, \dots, M$ **do**
- 3: 1) propose parameter $\theta' \sim q_\theta(\cdot | \theta)$
- 4: 2) run Algorithm 1 for R iterations using θ' . Keep last simulated network g'
- 5: 3) accept parameter θ' with probability

$$\alpha(\theta, \theta', g', g) = \min \left\{ 1, \frac{\exp\{\mathcal{Q}(g', X | \theta)\} p(\theta') q_\theta(\theta | \theta') \exp\{\mathcal{Q}(g, X | \theta')\}}{\exp\{\mathcal{Q}(g, X | \theta)\} p(\theta) q_\theta(\theta' | \theta) \exp\{\mathcal{Q}(g', X | \theta')\}} \right\}$$

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Simulation and Estimation in Finite Networks

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► what prior distribution $p(\theta)$?

► what proposal distribution $q_{\theta}(\cdot|\theta)$?

Simulation results

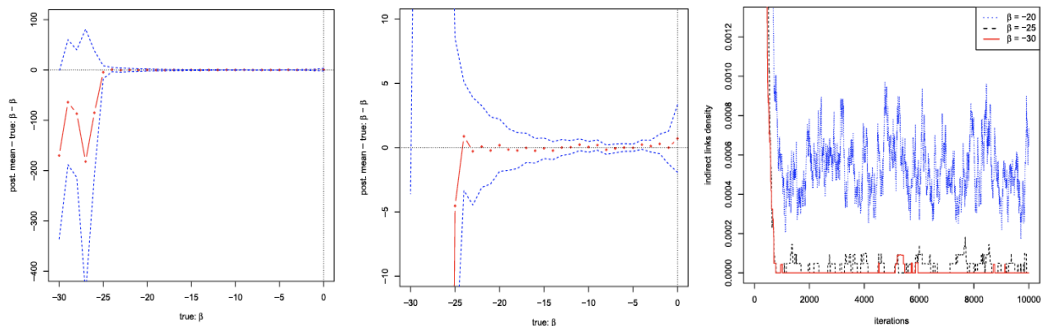


Figure: *Left:* Estimates of $\beta < 0$, with 95% credibility intervals (*middle:* zoom-in). *Right:* indirect links density.

- $\beta \geq 0 \implies$ Erdős-Rényi case, not identified
- $\beta < 0 \implies$ identified
- $\beta \ll 0 \implies$ estimation impossible: $\#$ indirect links close to 0

Simulation results

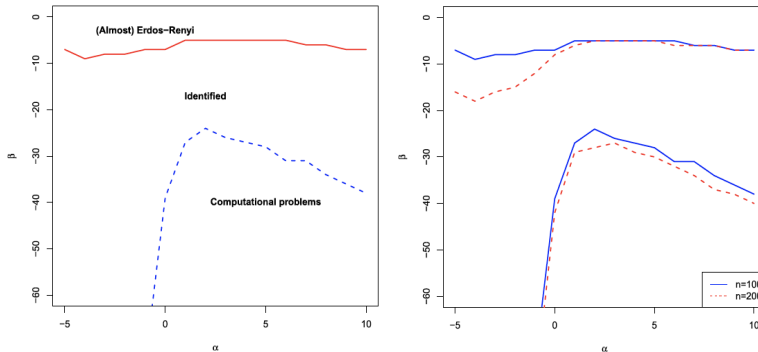


Figure: *Left:* approximate regions of identified parameters, for $n = 100$.
Right: comparison of regions for $n = 100$, $n = 200$.

Remark 4.

Regions of identified parameters (α, β) vary with n , the number of players.

Simulation results

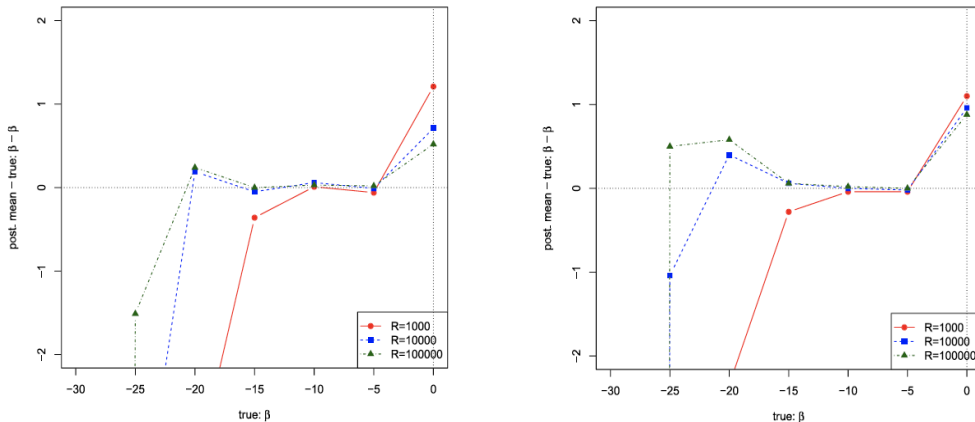


Figure: Difference between posterior estimates and true, for varying number of network simulations R : $n = 100$ (left) and $n = 200$ (right).

Simulation results

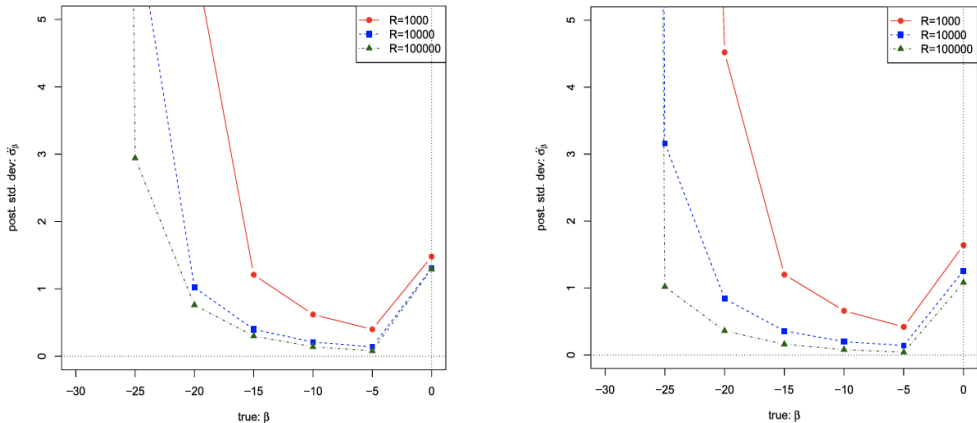


Figure: Posterior standard deviation for varying number of network simulations R : $n = 100$ (left) and $n = 200$ (right).

Simulation results

- ▶ $R = 1000 \implies$ imprecise estimates
- ▶ no significant difference between $R = 10,000$ and $R = 100,000 \implies$ suggest rule-of-thumb $R = 10,000$
- ▶ cost of increasing network simulations \implies almost linear $\mathcal{O}(R)$
- ▶ results suggest convergence is almost quadratic $\mathcal{O}(n^2)$ in this area of parameter space

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▶ what was the **computing time**?

▶ what about **real data applications**?

Conclusions

Conclusions

The paper in a nutshell:

- ❖ network formation model, combining **strategic** and **random networks** features
- ❖ payoffs depend on links: **direct** + indirect (**externalities**)
- ❖ **homogeneous** players meet sequentially at random, **myopically** updating links
- ❖ network formation process is a potential game and **converges to ERGM**, generating **directed dense** networks
- ❖ **identification**: only if at least **1 externality negative and sufficiently large**
- ❖ standard estimation for ERGMs exponentially slow \Rightarrow Bayesian MCMC (almost quadratic time)

Conclusions

Unclear points and questions:

- 👉 theoretical **quantification of “sufficiently large”** (negative) magnitude of β ?
- 👉 choice of prior for parameters $p(\theta)$?
- 👉 choice of proposal for network $q_g(\cdot|g)$?
- 👉 choice of proposal for parameters $q_\theta(\cdot|\theta)$?
- 👉 duration of **computing time** in simulations?
- 👉 **real data** applications?

Summary of discussions

▶ **11 october**

Mele (2017), *section 1-2*.

Discussant: Matteo Iacopini

▶ **18 october**

Mele (2017), *section 2, Appendix A* + Tamer (2003), *summary*.

Discussant: Carlo Santagiustina

▶ **25 october**

Mele (2017), *section 3*.

Discussant:




▶ **8 novemeber**

Mele (2017), *section 4*.







Discussant: Giulia Carallo

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